

Integration 11D

$$\begin{aligned}
 \text{1 a } y &= \ln |x^2 + 4| \\
 \Rightarrow \frac{dy}{dx} &= \frac{1}{x^2 + 4} \times 2x \quad (\text{chain rule}) \\
 \therefore \int \frac{x}{x^2 + 4} dx &= \frac{1}{2} \ln |x^2 + 4| + c
 \end{aligned}$$

$$\begin{aligned}
 \text{b } y &= \ln |e^{2x} + 1| \\
 \Rightarrow \frac{dy}{dx} &= \frac{1}{e^{2x} + 1} \times e^{2x} \times 2 \quad (\text{chain rule}) \\
 \therefore \int \frac{e^{2x}}{e^{2x} + 1} dx &= \frac{1}{2} \ln |e^{2x} + 1| + c
 \end{aligned}$$

$$\begin{aligned}
 \text{c } y &= (x^2 + 4)^{-2} \\
 \Rightarrow \frac{dy}{dx} &= -2(x^2 + 4)^{-3} \times 2x \quad (\text{chain rule}) \\
 \therefore \int \frac{x}{(x^2 + 4)^3} dx &= -\frac{1}{4} (x^2 + 4)^{-2} + c \\
 \text{or } -\frac{1}{4(x^2 + 4)^2} + c
 \end{aligned}$$

$$\begin{aligned}
 \text{d } y &= (e^{2x} + 1)^{-2} \\
 \Rightarrow \frac{dy}{dx} &= -2(e^{2x} + 1)^{-3} \times e^{2x} \times 2 \quad (\text{chain rule}) \\
 \therefore \int \frac{e^{2x}}{(e^{2x} + 1)^3} dx &= -\frac{1}{4} (e^{2x} + 1)^{-2} + c \\
 \text{or } -\frac{1}{4(e^{2x} + 1)^2} + c
 \end{aligned}$$

$$\begin{aligned}
 \text{e } y &= \ln |3 + \sin 2x| \\
 \Rightarrow \frac{dy}{dx} &= \frac{1}{3 + \sin 2x} \times \cos 2x \times 2 \quad (\text{chain rule}) \\
 \therefore \int \frac{\cos 2x}{3 + \sin 2x} dx &= \frac{1}{2} \ln |3 + \sin 2x| + c
 \end{aligned}$$

$$\begin{aligned}
 \text{f } y &= (3 + \cos 2x)^{-2} \\
 \Rightarrow \frac{dy}{dx} &= -2(3 + \cos 2x)^{-3} \times (-\sin 2x) \times 2 \\
 (\text{chain rule}) \\
 \therefore \int \frac{\sin 2x}{(3 + \cos 2x)^3} dx &= \frac{1}{4} (3 + \cos 2x)^{-2} + c
 \end{aligned}$$

$$\text{or } \frac{1}{4(3 + \cos 2x)^2} + c$$

$$\begin{aligned}
 \text{g } y &= e^{x^2} \\
 \Rightarrow \frac{dy}{dx} &= e^{x^2} \times 2x \quad (\text{chain rule}) \\
 \therefore \int x e^{x^2} dx &= \frac{1}{2} e^{x^2} + c
 \end{aligned}$$

$$\begin{aligned}
 \text{h } y &= (1 + \sin 2x)^5 \\
 \Rightarrow \frac{dy}{dx} &= 5(1 + \sin 2x)^4 \times \cos 2x \times 2 \\
 (\text{chain rule}) \\
 \therefore \int \cos 2x (1 + \sin 2x)^4 dx &= \frac{1}{10} (1 + \sin 2x)^5 + c
 \end{aligned}$$

$$\begin{aligned}
 \text{i } y &= \tan^3 x \\
 \Rightarrow \frac{dy}{dx} &= 3 \tan^2 x \times \sec^2 x \quad (\text{chain rule}) \\
 \therefore \int \sec^2 x \tan^2 x dx &= \frac{1}{3} \tan^3 x + c
 \end{aligned}$$

$$\begin{aligned}
 \text{j } \sec^2 x (1 + \tan^2 x) &= \sec^2 x + \sec^2 x \tan^2 x \\
 \therefore \int \sec^2 x (1 + \tan^2 x) dx &= \int \sec^2 x + \sec^2 x \tan^2 x dx \\
 &= \tan x + \frac{1}{3} \tan^3 x + c
 \end{aligned}$$

2 a $y = (x^2 + 2x + 3)^5$

$$\begin{aligned}\Rightarrow \frac{dy}{dx} &= 5(x^2 + 2x + 3)^4 \times (2x + 2) \\ &= 5(x^2 + 2x + 3)^4 \times 2(x + 1) \\ \therefore \int (x + 1)(x^2 + 2x + 3)^4 dx \\ &= \frac{1}{10}(x^2 + 2x + 3)^5 + c\end{aligned}$$

b $y = \cot^2 2x$

$$\begin{aligned}\Rightarrow \frac{dy}{dx} &= 2 \cot 2x \times (-\operatorname{cosec}^2 2x) \times 2 \\ &= -4 \operatorname{cosec}^2 2x \cot 2x \\ \therefore \int \operatorname{cosec}^2 2x \cot 2x dx &= -\frac{1}{4} \cot^2 2x + c\end{aligned}$$

c $y = \sin^6 3x$

$$\begin{aligned}\Rightarrow \frac{dy}{dx} &= 6 \sin^5 3x \times \cos 3x \times 3 \\ \therefore \int \sin^5 3x \cos 3x dx &= \frac{1}{18} \sin^6 3x + c\end{aligned}$$

d $y = e^{\sin x}$

$$\begin{aligned}\Rightarrow \frac{dy}{dx} &= e^{\sin x} \times \cos x \\ \therefore \int \cos x e^{\sin x} dx &= e^{\sin x} + c\end{aligned}$$

e $y = \ln |e^{2x} + 3|$

$$\begin{aligned}\Rightarrow \frac{dy}{dx} &= \frac{1}{e^{2x} + 3} \times e^{2x} \times 2 \\ \therefore \int \frac{e^{2x}}{e^{2x} + 3} dx &= \frac{1}{2} \ln |e^{2x} + 3| + c\end{aligned}$$

f $y = (x^2 + 1)^{\frac{5}{2}}$

$$\begin{aligned}\Rightarrow \frac{dy}{dx} &= \frac{5}{2} (x^2 + 1)^{\frac{3}{2}} \times 2x = 5x(x^2 + 1)^{\frac{3}{2}} \\ \therefore \int x(x^2 + 1)^{\frac{3}{2}} dx &= \frac{1}{5} (x^2 + 1)^{\frac{5}{2}} + c\end{aligned}$$

g $y = (x^2 + x + 5)^{\frac{3}{2}}$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{2} (x^2 + x + 5)^{\frac{1}{2}} \times (2x + 1)$$

$$\therefore \int (2x + 1) \sqrt{x^2 + x + 5} dx = \frac{2}{3} (x^2 + x + 5)^{\frac{3}{2}} + c$$

h $y = (x^2 + x + 5)^{\frac{1}{2}}$

$$\begin{aligned}\Rightarrow \frac{dy}{dx} &= \frac{1}{2} (x^2 + x + 5)^{-\frac{1}{2}} \times (2x + 1) \\ &= \frac{1}{2} \frac{(2x + 1)}{\sqrt{x^2 + x + 5}}\end{aligned}$$

$$\therefore \int \frac{2x + 1}{\sqrt{x^2 + x + 5}} dx = 2(x^2 + x + 5)^{\frac{1}{2}} + c$$

i $y = (\cos 2x + 3)^{\frac{1}{2}}$

$$\begin{aligned}\Rightarrow \frac{dy}{dx} &= \frac{1}{2} (\cos 2x + 3)^{-\frac{1}{2}} \times (-\sin 2x) \times 2 \\ &= -\frac{\sin 2x}{\sqrt{\cos 2x + 3}} \\ &= -\frac{2 \sin x \cos x}{\sqrt{\cos 2x + 3}} \\ \therefore \int \frac{\sin x \cos x}{\sqrt{\cos 2x + 3}} dx &= -\frac{1}{2} (\cos 2x + 3)^{\frac{1}{2}} + c\end{aligned}$$

j $y = \ln |\cos 2x + 3|$

$$\begin{aligned}\Rightarrow \frac{dy}{dx} &= \frac{1}{\cos 2x + 3} \times (-\sin 2x) \times 2 \\ &= -\frac{2 \sin 2x}{\cos 2x + 3} \\ &= -\frac{4 \sin x \cos x}{\cos 2x + 3} \\ \therefore \int \frac{\sin x \cos x}{\cos 2x + 3} dx &= -\frac{1}{4} \ln |\cos 2x + 3| + c\end{aligned}$$

3 a Let $I = \int_0^3 (3x^2 + 10x) \sqrt{x^3 + 5x^2 + 9} dx$

Consider $y = (x^3 + 5x^2 + 9)^{\frac{3}{2}}$

$$\frac{dy}{dx} = \frac{3}{2} (3x^2 + 10x) (x^3 + 5x^2 + 9)^{\frac{1}{2}}$$

$$\begin{aligned}\text{So } I &= \left[\frac{2}{3} (x^3 + 5x^2 + 9)^{\frac{3}{2}} \right]_0^3 \\ &= 486 - 18 = 468\end{aligned}$$

3 b Let $I = \int_{\frac{\pi}{9}}^{\frac{2\pi}{9}} \frac{6 \sin 3x}{1 - \cos 3x} dx$

Consider $y = \ln|1 - \cos 3x|$

$$\frac{dy}{dx} = \frac{3 \sin 3x}{1 - \cos 3x}$$

$$\begin{aligned} \text{So } I &= \left[2 \ln|1 - \cos 3x| \right]_{\frac{\pi}{9}}^{\frac{2\pi}{9}} \\ &= 2 \left(\ln \frac{3}{2} - \ln \frac{1}{2} \right) = 2 \ln 3 \end{aligned}$$

c Let $I = \int_4^7 \frac{x}{x^2 - 1} dx$

Consider $y = \ln|x^2 - 1|$

$$\frac{dy}{dx} = \frac{2x}{x^2 - 1}$$

$$\begin{aligned} \text{So } I &= \left[\frac{1}{2} \ln|x^2 - 1| \right]_4^7 \\ &= \frac{1}{2} (\ln 48 - \ln 15) \\ &= \frac{1}{2} \ln \frac{48}{15} = \frac{1}{2} \ln \frac{16}{5} \end{aligned}$$

d Let $I = \int_0^{\frac{\pi}{4}} \sec^2 x e^{4 \tan x} dx$

Consider $y = e^{4 \tan x}$

$$\frac{dy}{dx} = 4 \sec^2 x e^{4 \tan x}$$

$$\begin{aligned} \text{So } I &= \left[\frac{1}{4} e^{4 \tan x} \right]_0^{\frac{\pi}{4}} \\ &= \frac{1}{4} e^4 - \frac{1}{4} e^0 = \frac{1}{4} (e^4 - 1) \end{aligned}$$

4 Let $I = \int_0^k kx^2 e^{x^3} dx$

Consider $y = e^{x^3}$

$$\frac{dy}{dx} = 3x^2 e^{x^3}$$

$$\begin{aligned} \text{So } I &= \left[\frac{k}{3} e^{x^3} \right]_0^k \\ &= \frac{k}{3} (e^{k^3} - 1) = \frac{2}{3} (e^8 - 1) \\ k &= 2 \end{aligned}$$

5 Let $I = \int_0^{\theta} 4 \sin 2x \cos^4 2x dx$

Consider $y = \cos^5 2x$

$$\frac{dy}{dx} = -10 \sin 2x \cos^4 2x$$

$$\begin{aligned} \text{So } I &= \left[-\frac{2}{5} \cos^5 2x \right]_0^{\theta} \\ &= \left(-\frac{2}{5} \cos^5 2\theta \right) + \frac{2}{5} = \frac{4}{5} \\ \cos^5 2\theta &= -1 \Rightarrow \cos 2\theta = -1 \\ 2\theta &= \pi \Rightarrow \theta = \frac{\pi}{2} \end{aligned}$$

6 a $\int \cot x dx = \int \frac{\cos x}{\sin x} dx$

Consider $y = \ln|\sin x|$

$$\frac{dy}{dx} = \frac{\cos x}{\sin x}$$

$$\text{So } \int \cot x dx = \ln|\sin x| + c$$

b $\int \tan x dx \equiv \int \frac{\sin x}{\cos x} dx$

Consider $y = \ln|\cos x|$

$$\frac{dy}{dx} = -\frac{\sin x}{\cos x}$$

$$\begin{aligned} \text{So } \int \tan x dx &= -\ln|\cos x| + c \\ &\equiv \ln \left| \frac{1}{\cos x} \right| + c \\ &\equiv \ln|\sec x| + c \end{aligned}$$